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# Logic and Cognition: Two Faces of Psychologism

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**Abstract**

In this paper two concepts of psychologism in logic are outlined: the one which Frege and Husserl fought against and the new psychologism, or cognitivism, which underlies a cognitive turn in contemporary logic. Four issues such cognitively oriented logic should be interested in are indicated. They concern: new fields opened for logical analysis, new methods and tools needed to address these fields, neural basis of logical reasoning, and an educational problem: how to teach such logic? Several challenging questions, which arise in the context of these issues, are listed.

*Keywords*: Logic, reasoning, new psychologism, cognitivism.

## 1 Introduction

Logic emerged in Antiquity as an investigation of types of reasoning, both from the perspective of case-based analysis of their rationality, and from the perspective of their structures. So conceived, for many centuries logic stood in a close and natural relationship to the science of actual reasoning processes. As long as both logic and psychology were just parts of philosophy there was no real need for any precise demarcation. Boole [1854, p. 1] searched for "the fundamental laws of those operations of the mind by which reasoning is performed [and] some probable intimations concerning the nature and constitution of the human mind". For Mill [1858, p. 7] logic was the science of "both the processes itself of proceeding from known truths to unknown, and all intellectual operations auxiliary to this". For de Morgan [1847, p. 26] it was "the branch of inquiry [...] in which the act of the mind in reasoning is considered". Beneke [1832, p. 12] classified logic as the part of psychology that investigates relations between thinking and the reality. Thus when Erdmann [1870, vol. 3] coined the term 'psychologism' to describe Beneke's views it was merely a neutral description.

## 2 Psychologism: the repulsive face vs. the alluring one

Logic and cognition got 'divorced' (as Stenning and van Lambalgen [2008, p. 8–15] call it) mainly because of the antipsychologistic argumentation of

Gottlob Frege[1] and his proselyte Edmund Husserl. From the perspective of logic treated as a formal system, in the spirit of Fregean *Begriffsschrift*, the only interesting properties of actual reasoning are the objective ones: their structure, the relations between premises and a conclusion (propositions rather than sentences). Laws of logic are known *a priori*, they are not generalizations of experiences. Laws of logic refer to ideal objects, not to psychological entities. Actual thinking is not driven by the laws of logic [Husserl, 1900–1901, §§ 19–23]. Logic (and mathematics) is the most exact of all sciences, while psychology is imprecise and vague [Frege, 1884, p. 38]. The one has nothing to do with the other. After Frege and Husserl in formal logic the term 'psychologism' began to be associated negatively.

Achievements of this logic – call it formal, mathematical or symbolic – are enormous, especially when compared with a rather stable development of so-called traditional logic (from Aristotle to de Morgan). But the antimetaphysical enchantment by the Pure Form, so typical for Frege's pugnacious grandchildren (logical empiricists in particular) soon had to give place to a more realistic stance. Restoring the good name of the Truth by Tarski, Gödel's theorems, developments within philosophical logic and logical pragmatics are only a few steps towards inclusion into contemporary logic's area of interest, the problems of representation of structures of thought and language, that "go beyond the bare minimum provided by standard first-order logic" [van Benthem, 2008, p. 71]. It is important that the source of all this changes was reflection on a real thought and on a real language even if sometimes this source has been viewed as a bit embarrassing. The farther steps are marked by mutual infiltration of logic and Artificial Intelligence, in particular with respect to problem-solving, planning and diagnosis [Charniak and McDermott, 1985], and by granting a logical citizenship to an analysis of informational and heuristic value of fallacies [Van Eemeren *et al.*, 1996].

Nowadays we are witnessing a 'practical', or cognitive, turn in logic [Gabbay and Woods, 2005]. It does not declare results by Peano, Frege, Skolem or Tarski null and void. It claims that logic has much to say about actual reasoning and argumentation. Moreover, high standards of logical inquiry that we owe to Peano, Frege, Skolem, Tarski and others offer a new quality in research on reasoning and argumentation. Having in mind Corcoran's [1994] distinction of logic as formal ontology and logic as formal epistemology we may say that the aim of the practical turn is to make such formal epistemology even more epistemically oriented. This is not to say that this 'practically turned' (or cognitively oriented) logic becomes just a part of psychology. This is to say that this logic aquires a new task of "systematically keeping track of changing representations of information" [van Benthem, 2008, p. 73] and that it contests the claim that distinction be-

[1] Frege's work is the finial of the 'mathematical turn' in logic that was initiated by Leibniz [Gabbay and Woods, 2001].

tween descriptive and normative account on analysis of reasoning is disjoint and exhaustive [Gabbay and Woods, 2003, s. 37]. From different than a purely psychological perspective logic becomes – again – interested in answering Dewey's question: how we think? This is the new alluring face of psychologism (or cognitivism, as I prefer to call it) in logic, as opposed to the repulsive one, which Frege and Husserl fought against.

In my opinion there are at least four issues this cognitively oriented logic should be interested in. They are not of the same interest for every logician; nevertheless, all four are important if the renewed logical interest in actual human reasoning is to be considered as a serious one. These four issues concern: new fields opened for logical analysis, new methods and tools needed to address these fields, neural basis of logical reasoning, and, last but not least, an educational problem: how to teach such logic? All of them confront logic with exciting challenges and I am going to list some questions which arise in their context. Let us have a closer look at the four issues.

# 3 The four issues

## 3.1 New field

Most important proponents of the practical turn in logic emphasize that 'practicality' means first and foremost application of logic to the analysis of actual human reasoning:

> Logic is of course not experimental, or even theoretical, psychology, and it approaches human reasoning with purposes of its own. And a logical theory is not useless if people do not quite behave according to it. But the boundary is delicate. And I think the following should be obvious: if logical theory were totally disjoint from actual reasoning, it would be no use at all, for whatever purpose! [van Benthem, 2008, p. 69].

Two main problems arise in this context. First is the problem of application: what real (reasoning) cognitive processes, if any, are modelled by a given logical system? It is not the case that, if the answer is 'hard to say', the system in question is worthless. But it is more interesting from the cognitive point of view when such an aswer can be determined. However, one warning is in order here. It is well-known that the number of logical systems exceeds the number of stars in the sky (every modal logician will agree). It is probably pointless to try a jacket of every single system on a body of human cognition just to conclude 'it does not fit!'. If we are interested in question on what kind of logics real human reasoning is based, the second problem should be considered. This is the problem of extraction: Is it possible to extract underlying logics from our cognitive processes? Again, it would be rather pointless to attempt at such extraction from scratch: the problems of application and of extraction are interwined. Two short examples should clear the matter.

First example comes from a paper by Strannegård et al. [2010]. The authors conducted an experiment in which a random mix of 40 tautologies and 40 non-tautologies were presented to the participants, who were asked to determine which ones of the formulas were tautologies, with 45 s time-limit. On the basis of the results the authors propose a proof formalism for modelling propositional reasoning with bounded cognitive resources. They also define two particular proof systems for showing propositional formulas to be tautologies or non-tautologies.

What is really interesting is that the authors aimed at modelling real (or, to be more precise: as real as possible) reasoning processes, incorporating fundamental concepts and findings from cognitive psychology, concerning memory and reasoning processes, into their natural deduction style proofs. The resulting proof systems are expressed in an augmented language of Classical Propositional Calculus, extended to reflect reasoning processes involved in deciding if a given formula is a tautology or not (this is a kind of a 'language of thought' in the sense of Stenning and van Lambalgen [2008]). Rules of their systems aim at capturing, among others, observations of logical form of a formula, partial truth-functional evaluation of a formula, trading information for working memory space. Of particular interest is the following quotation from "Development Process" section of the paper:

> Our proof systems were developed on the basis of existing proof systems and cognitive models, interviews with the participants, think-aloud protocols, introspection, and experimental data [Strannegård et al., 2010, p. 300].

It is an open question whether this particular formalism is (cognitively) adequate. Nevertheless, it is clear that in process of such a development both application and extraction are interwined, providing feedback to each other.

The second example is even more instructive as it reveals how productive is transcending one-dimensional interpretations of experimental results on human reasoning. It is the problem of interpretation of Wason's selection task, as described by Stenning and van Lambalgen [2008, ch. 3]. In this well-known task a subject is presented with a set of four cards, labelled with letters and numbers (one typical set of labels is 'A', 'K', '4', '7'). The subject can see only the exposed face of cards and not the hidden back. On each card, there is a number on one of its sides and a letter on the other. The following rule is also presented to the subject:

> If ($p$) there is a vowel on one side, then ($q$) there is an even number on the other side.

The subject is informed that the rule applies only to the four cards and that the task is to decide which, if any, of these four cards must be turned in order to decide if the rule is true. The subject should not turn unnecessary cards.

If the conditional rule is interpreted as a material implication, then the correct solution of the task consist in applying *modus ponens* and *modus tollens* and the card that should be turned are the ones labelled with 'A' and '7'. The abstract, context-independent version of the task (as described above) yields typically 5% – 20% of correct answers. The results are almost reversed if the rule is construed in such a way that it concerns practical matters (like the rule 'if you drink alcohol here, you have to be over 18' in research conducted by Griggs and Cox [1982]).

But dialogue protocols quoted by the authors reveal that there is a number of pragmatic factors that affect solution of selection task in the abstract setting. Among them are: subject's understanding of truth and falsity of conditionals, descriptive vs. deontic interpretation of conditional, considering conditional as a rule which allows exceptions. In their fascinating analysis Stenning and van Lambalgen show that it is a misunderstanding just to claim that classically incorrect solutions to the selection task are simply irrational. They argue that:

> understanding [subject's] interpretation sometimes leads to clarification of what subjects are trying to do, and that often turns out to be quite different than the experimenter assumes [Stenning and van Lambalgen, 2008, p. 90].

The experimental data support the claim that humans are quite fluent 'practical logicians' [Riggs and Peterson, 2000; Scribner, 1997]. Much like a proficient tennis player, who is able to catch a difficult service ball and send it back to his opponent without much knowledge of geometry, we are able to apply different logics properly in different everyday contexts. We are capable of performing simple deductive inferences if needed, we can do reasonable abductions, even we are able to reason non-monotonically under the closed-world assumption. Of course, it is human to err in playing tennis as well as in reasoning. But still, several questions arise: If there are many different logics involved in our everyday reasoning processes, and if they are applicable in different tasks, and if we can switch smoothly between them, then maybe there exists a cognitive (meta)mechanism managing their applications? A mechanism for deciding, for example, which criteria for evaluating conclusions should be applied in a particular case? And maybe such mechanism can be formally modelled?

## 3.2 New methods and tools

One problem with applying logic to actual human reasoning is, that this reasoning often consists of non-verbal representations as premises and conclusion (consider geometrical proof without words [Nelsen, 1997] or different kinds of non-verbal abduction [Magnani, 2009]). Another problem is, that even more often reasoning rests on factors that are beyond reach of typical logical formalisms (like ordering premises in certain reasoning according to some pragmatic criteria of preference). Thus, as a result of expanding its area of interest, cognitively oriented logic faces the need of extending both the repertoire of its methods and the set of its tools.

An attractive direction of such an extension is connectionism. It is not because artificial neural networks (ANNs) are adequate models of human brain activity (they are not, as yet). What is really enticing is that ANNs allow for modelling phenomena which escape symbolic languages.

A good example is the role of emotions in abductive reasoning. Thagard [2006, 2007] argues that every serious theory of abduction must take emotional context of this kind of reasoning into account. On the one hand, abduction is triggered by emotions: we start abducing when we encounter surprising phenomena that are worth to be accounted for (and Magnani [2009] argues that 'we' has a broader meaning than 'we, humans'). On the other hand, abductive finale is also of emotional character: if we manage to make sense of puzzling facts the result is satisfaction (consider typical endings of Holmesian detective stories, great examples of employing abduction in solving mysteries). There are some prospects concerning modelling emotions via symbolic logic [Adam *et al.*, 2009]. Nevertheless, models employing ANNs look more convincing [Eliasmith, Thagard, 2001; Thagard, Litt, 2008]. Connectionist logics already open a new dimension in proof theory [d'Avila Garcez *et al.*, 2008], but this novelty is of quantitative character, similar to the one offered by a new proof technique. An open and interesting question is if connectionist logics may offer also qualitative novelty, resulting in something comparable to intensional revolution in logic.

Another attractive direction of extension of methods and tools of logic is application of Labelled Deductive Systems [Gabbay, 1996]. Although the idea of using labels in logic is not new, Gabbay is right in stressing that it is new to consider the labelling as part of the logic [Gabbay, 1996, p. 12, footnote 5]. Again, abduction is a good example. One step in such a reasoning consists in evaluation of generated hypotheses against certain predefined criteria. In a realistic stance this means more than just checking consistency or logical dependencies between hypotheses and background theories. There are some pragmatic criteria that have to be taken into account, like Peircean economy, testability or explanatory power. In the setting of Labelled Deductive Systems such criteria and evaluation of hypotheses against them may be embedded directly into inference mechanisms, this time of a symbolic character.

### 3.3 Neural basis

Third issue for cognitively oriented logic is the neural basis of logical reasoning. It is not just a neuroimaging problem of what parts of the brain are responsible for performing such operations. This question is probably of interest, but not that much for logicians. Much more interesting question is if logical reasoning is performed by the same parts of the brain that other kinds of reasoning? In an fMRI study Monti *et al.* [2009] compared logical inferences relying on sentential connectives (like: not, or, if ... then) to linguistic inferences based on syntactic transformation of sentences involving ditransitive verbs (like: give, say, take). The results indicate that logical inference is not embedded in natural language. Thus further questions arise: What about logical vs. mathematical reasoning? And what about different logics? Are regions of the brain recruited in epistemic or deontic inferences the same as in 'classical' sentential inferences? Are erotetic inferences (that is, inferences involving questions) processed by the same regions that declarative ones? And what about non-verbal logical inferences?

### 3.4 Educational concern

A fundamental educational problem is that to make sense of the interplay of logic and psychology one needs a substantial competence in logic. For a student this means going trough the foundations of set theory, model theory, classical and some non-classical systems and their metatheory before he or she will be able to grasp the idea of even the basic applications of logical analysis to reasoning processes. And this way may be seen as quite a trying one. Thus, in teaching logic, how to avoid Scylla of trivial narrative without proper formal basis and Charybdis of excessively hermetic formalism? How not to reduce logic neither to critical thinking exercises, nor to formal mindteasers, a kind of mind fitness for our students? How to teach logic as both formally and empirically grounded science of reasoning processes? How to design a cognitively oriented course in logic, which is a subject of secondary importance in a typical curriculum? This is probably the most practical and challenging problem of all I mentioned in this paper.

## 4 Closing remarks

The practical turn does not create a rival for the mathematical logic. It forms a next step in the development of logic which results in inclusion of some areas of cognitive science, psychology and computer science into its hard core. Consequently, logic becomes capable of modelling actual cognitive activity of real life agents. Thus, as Gabbay and Woods [2001, s.

141] put it, "whereas mathematical logic must eschew psychologism, the new [that is, cognitively oriented] logic cannot do without it": this new psychologism, or cognitivism, constitutes the essence of logic so conceived. A paraphrase of Einstein's famous formulation may serve as its catchword: in analysis of reasoning psychology without logic is lame, whereas logic whitout psychology is blind.

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