Search and Check. Problem Solving by Problem Reduction

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(Paweł Łupkowski and Mariusz Urbański were
supported by Foundation for Polish Science.)
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Abstract. In this paper we outline a goal-directed abductive problem-solving procedure, developed within the framework of Inferential Erotetic Logic. This procedure is considered to be the core procedure of a reasoning engine for multi-layer self-organizing systems, which will, in particular, automatically manage reconfiguration of components in a given system in a safe and optimal way and its adaptation.

1 Introduction

As more and more sophisticated knowledge-based systems appear the automation of different aspects of their activity becomes crucial. When such knowledge-based systems are considered to be intelligent their rational behaviour is designed as goal-directed, thus, simulating one of the fundamental characteristics of human reasoning. The crucial role here is traditionally allocated to problem-solving techniques. The natural approach to the automation of the process of solving complex problems is often described as follows: an initial problem is decomposed into sub-problems with subsequent attempts to solve these (presumably) simpler sub-problems, and then the solutions to these sub-problems are assembled into an overall solution to the original problem. The underlying ideology of this idea which has its roots in ancient philosophy is transparent. By decomposing a problem we aim at uncovering subordinated problems for which the solutions are much easier or even trivial. Yet, despite the maturity of this setting which is as old as the conventional artificial intelligence, none of the techniques developed in computer science and artificial intelligence have led to more or less efficient implementation of this framework of problem-solving by problem reduction. In the best case, developed tools can solve some designated classes of area-specific problems (e.g. theorems of mathematics, geometry, etc) and their obvious weakness is dependence on the nature of the problems considered.

In this paper we outline a goal-directed abductive problem-solving procedure, developed within the framework of Inferential Erotetic Logic (IEL).

The Inferential Erotetic Logic (IEL) ([17], [19]) is a useful tool in the area of analyzing and modelling such components of intelligent activity as planning, problem solving, and searching for information in large data/knowledge bases [18]. Important developments within the framework of IEL are Erotetic Search Scenarios (ESS) [20] and Socratic Proofs (SP) [21]. ESS are based on the idea of providing conditional instructions for solving an initial problem, informing us which questions should be asked and when they should be asked. Moreover, an erotetic search scenario shows where to go if a direct answer to a query appears to be acceptable and does so with respect to any direct answer to each query. SP is a very specific technique, which reduces the complexity of standard problem-solving methods by using pure questioning only. ESS, Synthetic Tableaux Method (STM — a semantically-motivated goal-directed proof method based on ESS [11]) and SP have been already applied to a number of logics (propositional and first-order) ([8],[9],[10],[12],[15],[21],[22]), and have also been implemented ([6], [7], [13], [14]).

Our ultimate aim is to design and implement an intelligent system which takes as an input an initial problem represented in some high level specification language such as a logical language and in order to solve it automatically applies certain problem reduction techniques. However, unlike other approaches to problem-solving, we base our project on completely new, logical foundations. Treating problems as questions, we start with a question formulated in a certain logical language and propose to organize problem-solving as finding an answer to the question.

2 System Architecture

The underlying architecture is thought of as a hybrid system involving ***erotetic search*** and ***proof search engines*** defined over some initial ***knowledge base***.

The erotetic search engine is responsible for creating an erotetic search scenario for an initial question: the system synthesizes the answer as its output to this question on the basis of a given set of initial premises and by means of asking and answering auxiliary questions. This search engine is supposed to be defined over various contexts — classical, modal, temporal, paraconsistent, intuitionistic, thus tackling of different types of questions. The distinguished feature of our approach is that it enables us at certain stages of current search to use previously obtained scenarios.

The proof search engine is considered being based on goal directed procedures and being responsible for establishing various required deductive inferences, for example, between the components of the declarative part of the knowledge base and possible answers to the question, or between answers to different questions. It can also be used for checking the knowledge base for consistency.

Similar to the erotetic search engine, here we suggest to incorporate various goal directed automated theorem proving methods developed for classical and non-classical logics. Again, similar to the erotetic search engine, we plan to develop a technique which enables at certain stages of current search the embedding of previously obtained proof search schemes.

The knowledge base (KB) contains declarative part, stores achieved erotetic search scenarios and proof search schemes. The declarative part of the knowledge base can be of a different nature: it can be specified in the language of classical logic (propositional or first-order), or can be extended by modal or temporal contexts, on the one hand, or inconsistent and/or incomplete contexts, on the other hand.

The functionality of the system is thought in the coordinated activity of these components which we plan to develop as independent cooperating intelligent agents.

3 The Procedure

We consider an initial problem to be expressed by an interrogative of the form
(*) $?\{A_1, \ldots, A_n\}$

where $A_1, \ldots, A_n$ are conceptually accessible solutions (possibly organized according to decreasing order of preference).

By X we represent an initial KB.

We define the following search procedure:

Step 1.

for every $i = 1, \ldots, n$
check if:
$X \vdash A_i$
$X \vdash \neg A_i$

1. for the lowest i for which $X \vdash A_i$ (but $X \nvdash \neg A_i$), STOP (A_i is the most preferred solution which is derivable from KB);
2. if for every i: $X \vdash \neg A_i$, then STOP (there is no solution for the problem obtainable from the current KB);
3. if there is no i such that $X \vdash A_i$ and for at least one j: $X \nvdash A_j$ and $X \nvdash \neg A_j$ (A_j is unresolved), then go to step 2.

Step 2.

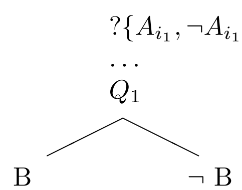
Make list
(**) $A_{i_1}, \ldots, A_{i_m}$

of all the unresolved solutions (possibly organized according to decreasing order of preference) and go to Step 3.

Step 3.

for A_{i_1}:

1. ask question of the form $?\{A_{i_1}, \neg A_{i_1}\}$
2. build an analytic erotetic search scenario for $?\{A_{i_1}, \neg A_{i_1}\}$ to the level of the first query:



$Q_1 = ?\{B, \neg B\}$

   (according to the definition of analytic ESS, all the queries of such scenario are simple yes-no questions, see [20])
3. check if:
   $X \vdash B$
   $X \vdash \neg B$
   (i) if $X \vdash B$ or $X \vdash \neg B$, then continue to the level of the next query (on each path);
   (ii) if $X \nvdash B$ and $X \nvdash \neg B$, then delete both B and $\neg B$ and embed an analytic search scenario for Q_1;
4. for every query repeat Step 3 1.–3.
5. If:
   (i) a path is obtained such that it leads to A_{i_1} and answers to all its queries are derivable from X, then STOP (A_{i_1} is the most preferred solution which is derrivable from KB);
   (ii) a path is obtained such that it leads to $\neg A_{i_1}$ and answers to all its queries are derivable from X, then GO TO A_{i_2};
   (iii) no path is such that all its queries are derivable from X, then:
      (a) if all the paths lead to $\neg A_{i_1}$, then GO TO A_{i_2};
      (b) if there exist paths leading to A_{i_1}, then mark them: ABD and GO TO A_{i_2}.

   Repeat Step 3 1.–5. for $A_{i_2}, \ldots A_{i_m}$ of (**)

   Step 4.

If in Step 3. STOP is not obtained, then:

1. if there is no path marked ABD, then STOP (the problem has no solution that can be reached on the basis of KB);
2. for every A_{j_k} such that there exists at least one path leading to A_{j_k} and marked ABD: if $e_{j_1}, \ldots, e_{j_h}$ are all the ABD-paths for A_{j_k}, then generate all the abductive explanations for A_{j_k} on the basis of $e_{j_1}, \ldots, e_{j_h}$ and STOP.

Abductive explanations obtained in Step 4.2 indicate information that needs to be added to X in order to obtain each particular solution.

An example of abductive explanations generating procedure can be found in [16].

4 Conclusions and Further Work

The procedure outlined in the present paper has many advantages. The most important are intuitiveness and generality (it is not dependent on the nature of the problems considered). We hope to apply this procedure as a core of a reasoning engine for multi-layer self-organizing systems, which will automatically manage reconfiguration of components in a given system in a safe and optimal way.

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